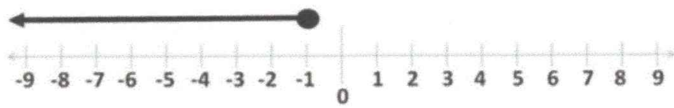
Mr. Michael T. Davis Pre-Calculus	Practice Final Exam Unit 1 through Unit 5 May 18, 2017

Name: Solutions / Answers

1. Write the domain of the graph in both inequality and interval notation.



Inequality: $x \le -1$ Interval: $(-\infty, -1]$

2. Write the domain of the graph in both inequality and interval notation.

Inequality: $-2 \le x < 5$ Interval: $[-2, 5)$

3. Solve each inequality. Graph the solution set on a number line, AND write the solution in interval notation. *

a. $-1 < \frac{1}{2}p + 4 \le 8$

$-5 < \frac{1}{2}p \le 4$

$-10 < p \le 8$

$(-10, 8]$

b. $4 - 2x > 16$ *or* $3x - 10 > 11$

$-2x > 12$ or $3x > 21$

$x < -6$ or $x > 7$

$(-\infty, -6) \cup (7, \infty)$

4. Graph the line with equation $4x - 3y = 12$

If $x = 0$: $4(0) - 3y = 12$

$-3y = 12$

$y = -4$

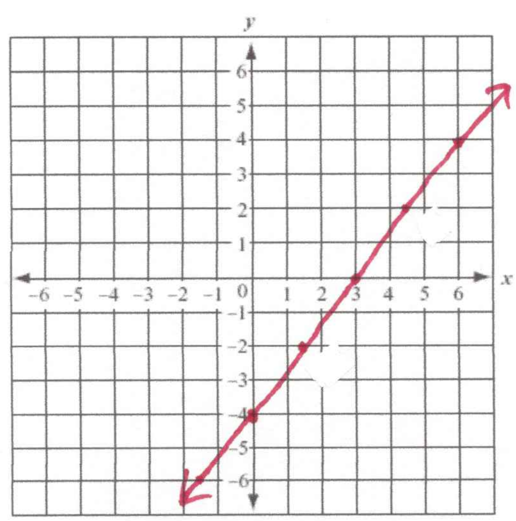
$(0, -4)$

If $y = 0$: $4x - 3(0) = 12$

$4x = 12$

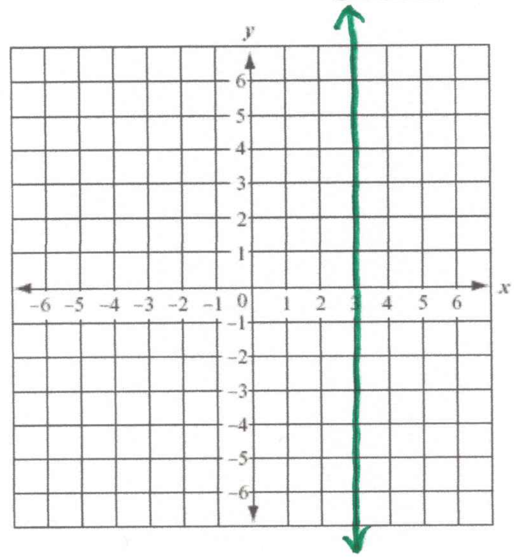
$x = 3$

$(3, 0)$



5. Write an equation for the line shown.

$x = 3$



6. Given a line containing the points $(-2,5)$ and $(-4,2)$. Find an equation of the line in slope-intercept form. Convert your equation to standard from.

$m = \frac{2-5}{-4--2} = \frac{-3}{-2} = \frac{3}{2}$ $\quad y = \frac{3}{2}x + b \quad (-4,2) \quad 2 = \frac{3}{2}(-4) + b$

$2 = -6 + b \quad 8 = b \quad y = \frac{3}{2}x + 8 \quad 2y = 3x + 16$ ✓

$-3x + 2y = 16$

7. Simplify $49^{\frac{1}{2}}$

$49^{\frac{1}{2}} = \sqrt{49} = 7$

8. Simplify $27^{\frac{2}{3}} =$

$27^{2/3} = (27^{\frac{1}{3}})^2 = (\sqrt[3]{27})^2 = (3)^2 = 9$

9. $\sqrt[4]{16} =$

$\sqrt[4]{16} = 2$

10. $\frac{35x^4y^8}{7x^{-3}y^5} =$ $\quad 5x^7y^3$

11. $8x^2y^6 + x^2y^6 =$ $9x^2y^6$

12. $(4xy^5)^3 =$ $4^3x^3y^{15} = 64x^3y^{15}$

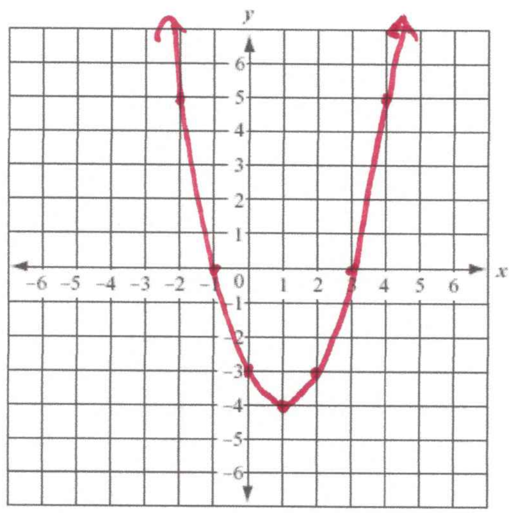
13. $(8x^2y^6)(x^2y^6) =$ $8x^4y^{12}$

14. Square the binomial and simplify $(x-7)^2$ $= (x-7)(x-7) = x^2 - 14x + 49$

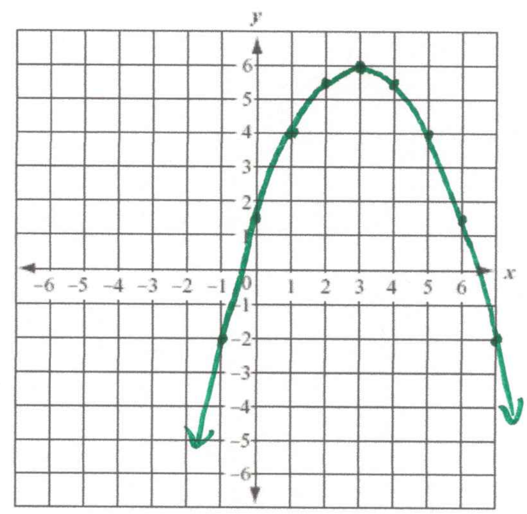
15. Multiply and simplify $(3x-1)(4x+5)$ $= 12x^2 + 15x - 4x - 5$

$= 12x^2 + 11x - 5$

16. Graph $f(x) = (x+1)(x-3)$



17. Graph $f(x) = -\frac{1}{2}(x+3)^2 + 6$



18. Given the parent function $f(x)=x^2$, circle the letter of the correct description of the transformation $T(x)=(x-3)^2+1$.

a. Vertical shift up 3 units and horizontal shift right 1 unit

b. Horizontal shift right 3 units and vertical shift up 1 unit

c. Horizontal stretch by a factor of 3 and vertical shift down 1 unit

d. Horizontal compression by a factor of 3 units and vertical shift down 1 uni

19. Line A has equation $y=\frac{2}{5}x-6$. Line B contains the point $(4,-9)$ and is perpendicular to line A. Determine an equation in any form for line B.

$y=-\frac{5}{2}x+b \quad -9=-\frac{5}{2}(4)+b \quad -9=-10+b \quad 1=b$

$y=-\frac{5}{2}x+1$

20. First, convert $f(x)=3x^2-4x-15$ to factored form, then determine the coordinates of the x-intercepts of the parabola with this equation.

$f(x)=(3x+5)(x-3) \qquad 0=(3x+5)(x-3)$

$3x+5=0 \qquad x-3=0 \qquad \left(-\frac{5}{3},0\right) \;\&\; (3,0)$

$x=-\frac{5}{3} \qquad x=3$

21. Given functions $f(x)=x^2-7$ and $g(x)=x-3$, evaluate $f(g(x))$

$f(g(x))=f(x-3)=(x-3)^2-7 = x^2-6x+9-2$

$= x^2-6x+7$

22. Factor $x^2-6x-40$ $=(x-10)(x+4)$

23. Factor $3x^2-11x-4$ $=(3x+1)(x-4)$

24. Factor $5y^2+12y+7$ $= (5y+7)(y+1)$

25. Factor $9k^2-49$ $= (3k-7)(3k+7)$

26. Factor $n^2+8n+16$ $= (n+4)(n+4) = (n+4)^2$

27. Factor $5m^3-45m$ $= 5m(m^2-9) = 5m(m-3)(m+3)$

28. Factor $3y^3+6y^2-45y$ $= 3y(y^2+2y-15) = 3y(y+5)(y-3)$

29. Factor z^4-13z^2+36 $= (z^2-4)(z^2-9) = (z-2)(z+2)(z-3)(z+3)$

30. Factor $x^3+5x^2+4x+20$ using grouping

$x^2(x+5)+4(x+5) = (x+5)(x^2+4$

31. Factor $x^3 - 7x^2 - 4x + 28$ using grouping

$$x^2(x-7) - 4(x-7) = (x-7)(x^2-4)$$
$$= (x-7)(x-2)(x+2)$$

32. Simplify $\frac{w+1}{16} + \frac{w-2}{20}$ $= \frac{5\cdot(w+1)}{5(16)} + \frac{4(w-2)}{4(20)} = \frac{5w+5}{80} + \frac{4w-8}{80}$

$$= \frac{9w-3}{80}$$

33. Simplify $\frac{2x}{x-5} - \frac{3}{x}$ $= \frac{x(2x)}{x(x-5)} - \frac{(x-5)(3)}{(x-5)(x)} = \frac{2x^2}{x^2-5x} - \frac{3x-15}{x^2-5x}$

$$= \frac{2x^2-3x+15}{x^2-5x}$$

For #'s 27-35, solve for real and imaginary solutions, give all irrational solutions in simplified radical form, and give all imaginary numbers in i-form.

34. Solve $(x-5)(3x+2) = 0$

$x-5=0$ $\quad$ $3x+2=0$

$x=5$ $\quad$ $3x=-2$

$x = \frac{-2}{3}$

$\{5, \frac{-2}{3}\}$

35. Solve $x^2 + 7x = 0$

$x(x+7)=0$

$x=0$ $\quad$ $x+7=0$

$x=-7$ $\quad$ $\{0, -7\}$

36. Solve $x^2 - 16 = 0$ $\quad$ $(x-4)(x+4)=0$

$x-4=0$ $\quad$ $x+4=0$

$x=4$ $\quad$ $x=-4$ $\quad$ $\{4, -4\}$

37. Solve $x^2+100=0$

$x^2=-100$ $\sqrt{x^2}=\pm\sqrt{-100}$ $x=\pm i\sqrt{100}$

$x=\pm i\cdot 10$ $x=\pm 10i$ $\{10i, -10i\}$

38. Solve $2y^3-24y=0$

$2y(y^2-12)=0$ $2y=0$ $y^2-12=0$

$y=0$ $y^2=12$

$y=\pm\sqrt{12}$

$\{0, 2\sqrt{3}, -2\sqrt{3}\}$ $y=\pm 2\sqrt{3}$

39. Solve $4x^2-25=0$

$(2x-5)(2x+5)=0$ $2x-5=0$ $2x+5=0$

$2x=5$ $2x=-5$

$x=\frac{5}{2}$ $x=\frac{-5}{2}$

$\{\frac{5}{2}, \frac{-5}{2}\}$

40. Solve $3x^2-4x-20=0$

$(3x-10)(x+2)=0$

$3x-10=0$ $x+2=0$

$3x=10$ $x=-2$

$x=\frac{10}{3}$ $\{\frac{10}{3}, -2\}$

41. Solve $x^2-4x+13=0$ using the quadratic formula

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)}=\frac{4\pm\sqrt{16-52}}{2}$$

$$x=\frac{4\pm\sqrt{-36}}{2}=\frac{4\pm 6i}{2}=2\pm 3i$$

$\{2+3i, 2-3i\}$

42. Solve $m^4-29m^2+100=0$

$(m^2-4)(m^2-25)=0$

$(m-2)(m+2)(m-5)(m+5)=0$

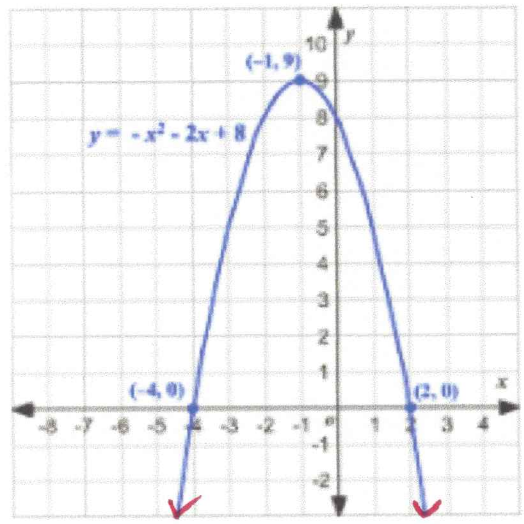
$\{2, -2, 5, -5\}$

43. Solve $\frac{10}{x+2}=\frac{15}{x+3}$ by multiplying the LCD on both sides

$\frac{2}{x+2}=\frac{3}{x+3}$ $2(x+3)=3(x+2)$ $2x+6=3x+6$

$2x=3x$ $0=x$ $\{0\}$

44. Given the graph of $y = f(x)$ above, determine the following (approximate if necessary):



a. Write the domain of $y = f(x)$ in both notations.

$(-\infty, \infty)$ $\qquad -\infty < x < \infty$

b. Write the range of $y = f(x)$ in both notations.

$(-\infty, 9]$ $\qquad y \leq 9$

c. Determine the value of $f(0)$

$f(0) = 8$

d. Determine the value(s) of x for which $f(x) = 5$

$x = -3$ and $x = 1$

e. Using either notation, determine the value(s) of x for which $f(x) = 0$ equal

$x = -4$ and $x = 2$

f. Using either notation, determine the values of x for which $f(x)$ is decreasing

$(-1, \infty)$ $\qquad x > -1$

45. The rational function $f(x)=\dfrac{x^2-9}{x^2-4x-21}$ has the following domain:

a. $D:\mathbb{R}, x \neq 3, x \neq -3$ b. $D:\mathbb{R}, x \neq -7, x \neq 3$ c. $D:\mathbb{R}, x \neq 7, x \neq -3$

d. $D:\mathbb{R}, x \neq -4, x \neq -21$ e. $D:\mathbb{R}$

$(x-7)(x+3) \neq 0 \quad x \neq 7$
$x \neq -3$

46. The rational function $f(x)=\dfrac{x-8}{x^2-25}$ has an x-intercept at:

a. $y=0$ b. $y=8$ c. $x=8$

d. $y=\dfrac{8}{25}$ e. There is no x-intercept

The numerator is zero at $x=8$ therefore $f(8)=0$ $(8,0)$

47. The rational function $f(x)=\dfrac{10}{x^2-4}$ has a horizontal asymptote with equation:

a. $y=0$ b. $x=2$ c. $y=10$

d. $x=-2$ & $x=2$ e. There is no horizontal asymptote

the degree of the denominator is greater

48. The rational function $f(x)=\dfrac{6x^2+3x-1}{2x^2-18}$ has a horizontal asymptote with equation:

a. $y=6$ b. $x=3$ c. $y=0$

d. $y=3$ e. None of the above

$y=\frac{6}{2}=3$

49. The rational function $f(x)=\dfrac{x^2-8x+16}{(x-4)(x+4)}$ has a hole at:

a. $y=1$ b. $y=0$ c. $x=-4$

d. $x=4$ e. There is no hole

$\dfrac{(x-4)(x-4)}{(x-4)(x+4)}$

50. The rational function $f(x)=\dfrac{x-18}{x^2-9}$ has a y-intercept at:

a. $y=3$ b. $x=18$ c. $x=9$

d. $y=2$ e. There is no y-intercept

$x=0 \quad f(0)=\dfrac{0-18}{0^2-9}=\dfrac{-18}{-9}=2 \quad y=2$

51. The rational function $f(x)=\frac{x^2+7x+10}{x-2}$ has a vertical asymptote with equation:

a. $x=2$ & $x=5$
b. $x=-5$ only
c. $x=-2$ & $x=-5$
d. $x=2$ only
e. There is no vertical asymptote

Denominator is equal to zero at x=2 and the numerator is not

52. Without a calculator evaluate $\log 1000=$ 3

53. Without a calculator evaluate $\log_5 1=$ 0

54. Without a calculator evaluate $\log_8 0=$ undefined

55. Without a calculator evaluate $\log_3 \frac{1}{9}=$ $= -2$

56. Without a calculator evaluate $\log_2 32=$ 5

57. Without a calculator evaluate $\log(-100)=$ undefined

58. Expand $\ln(x\cdot y^2)=$ $\ln x+\ln y^2 = \ln x + 2\ln y$

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59. Expand $\log\left(\dfrac{m}{n^3}\right) =$ $\log m - \log n^3 = \text{lom} - 3\log n$

60. Evaluate by condensing $\log 4 + 2\log 5 =$ $\log 4 + \log 5^2 = \log 4 + \log 25 =$

$\log(4 \cdot 25) = \log 100 = 2$

61. Without a calculator evaluate $\ln\left(e^4\right) =$ 4

62. Without a calculator evaluate $\log\left(10^5\right) =$ 5

63. With a calculator evaluate $\log_4 12 =$ $\dfrac{\log 12}{\log 4} \approx 1.7925$

Also $\dfrac{\ln 12}{\ln 4} \approx 1.7925$

64. Solve for x: $\log_x 64 = 2$

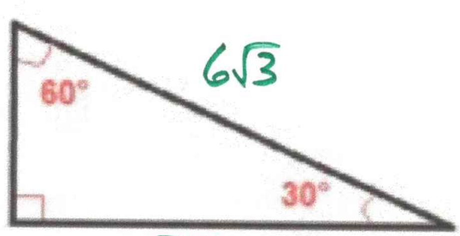
$x^2 = 64$ $x = 8$ $x \neq -8$ the base cannot be negative

65. Solve for x: $\ln x = 2$ $\log_e x = 2$ $e^2 = x$ $x = e^2$

66. Without a calculator solve $6^x = 12$ $\log 6^x = \log 12$ $x \log 6 = \log 12$

$x = \dfrac{\log 12}{\log 6}$

67. Given the special triangle shown with $6\sqrt{3}$ cm as its hypotenuse length, find the lengths of the other two sides.

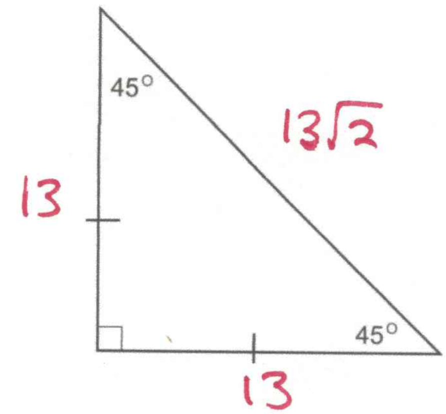


$6\sqrt{3}$

$3\sqrt{3}$

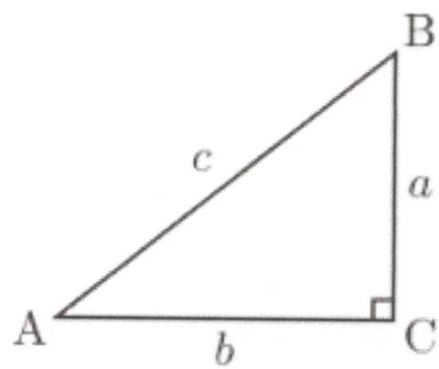
$3\sqrt{3}\sqrt{3} = 9$

68. Given the special triangle shown with $13\sqrt{2}$ cm as its hypotenuse length, find the lengths of the other two sides.



69. Evaluate $\cos B = \frac{a}{c}$

70. Evaluate $\tan A = \frac{a}{b}$



71. Evaluate $\csc B = \frac{c}{b}$

72. Evaluate $\sin A = \frac{a}{c}$

73. Evaluate $\sec A = \frac{c}{b}$

74. Evaluate $\cot B = \frac{a}{b}$

75. Complete the statement (fill in the blank): $\sin \underline{52^\circ} = \cos 38^\circ$

76. Convert $\frac{\pi}{6}$ to an angle measure in degrees.

$$\frac{\pi}{6} = \frac{180^\circ}{6} = 30^\circ$$

77. Convert 45° to an angle measure in radians,

$$45^\circ = 45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

78. Convert $\frac{\pi}{2}$ to an angle measure in degrees.

$$\frac{\pi}{2} = \frac{180^\circ}{2} = 90^\circ$$

79. Convert 240° to an angle measure in radians.

$$240^\circ = 240^\circ \cdot \frac{\pi}{180} = \frac{4\pi}{3}$$

80. Convert $\frac{7\pi}{6}$ to an angle measure in degrees.

$$\frac{7\pi}{6} = \frac{7(180^\circ)}{6} = 7 \cdot 30^\circ = 210^\circ$$

81. Convert 315° to an angle measure in radians.

$$315^\circ = 315^\circ \cdot \frac{\pi}{180^\circ} = \frac{7\pi}{4}$$

82. Write $\sin\frac{7\pi}{4}$ in terms of a reference angle (answer with radians)

$$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4}$$

83. Write $\cos 150^\circ$ in terms of a reference angle (answer with degrees)

$$\cos 150^\circ = -\cos 30^\circ$$

84. Write $\sin 240^\circ$ in terms of a reference angle (answer with degrees)

$$\sin 240^\circ = -\sin 60^\circ$$

85. Write $\tan\frac{2\pi}{3}$ in terms of a reference angle (answer with radians)

$$\operatorname{Tan}\frac{2\pi}{3} = -\operatorname{Tan}\frac{\pi}{3}$$

86. Evaluate the trigonometric expression $\sin\frac{\pi}{6} =$ $\frac{1}{2}$

87. Evaluate the trigonometric expression $\tan\frac{\pi}{4} =$ 1

88. Evaluate the trigonometric expression $\csc 150^\circ =$ $\frac{1}{\sin 150^\circ} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

89. Evaluate the trigonometric expression $\tan\frac{\pi}{2} =$ $\frac{1}{0}$ = undefined

90. Evaluate the trigonometric expression $\sec\frac{2\pi}{3} =$ $\frac{1}{\cos\frac{2\pi}{3}} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$

91. Evaluate the trigonometric expression $\cos 0 =$ 1

92. Evaluate the trigonometric expression $\sin\frac{\pi}{2} =$ 1

93. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ 60° or $\frac{\pi}{3}$

94. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ $45°$ or $\frac{\pi}{4}$

95. $\sin^{-1}(0) =$ 0 or $0°$

96. $\cos^{-1}(-1) =$ $180°$ or π

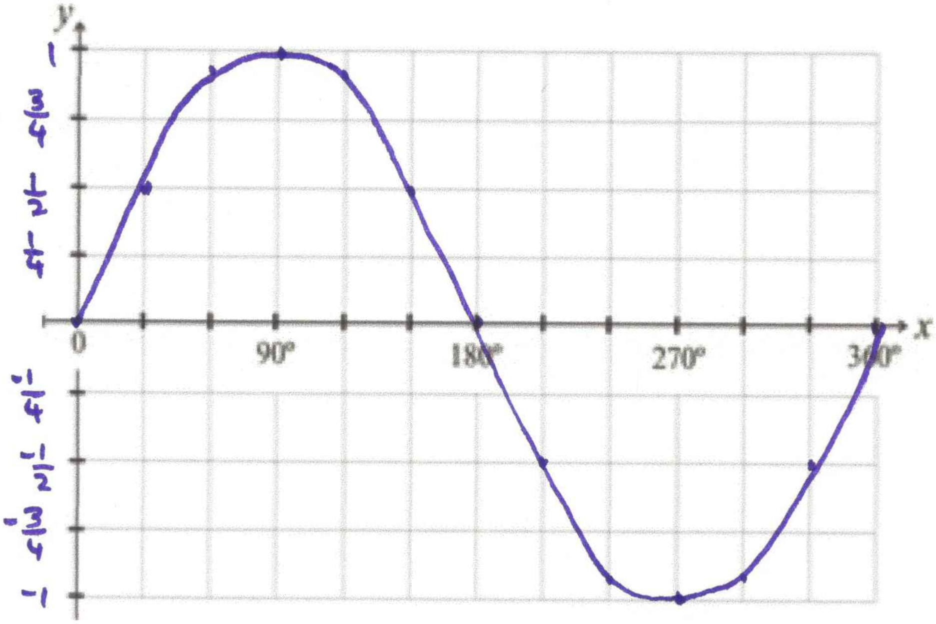
97. $\tan^{-1}(0) =$ 0 or $0°$

98. $\tan^{-1}(\sqrt{3}) =$ $60°$ or $\frac{\pi}{3}$

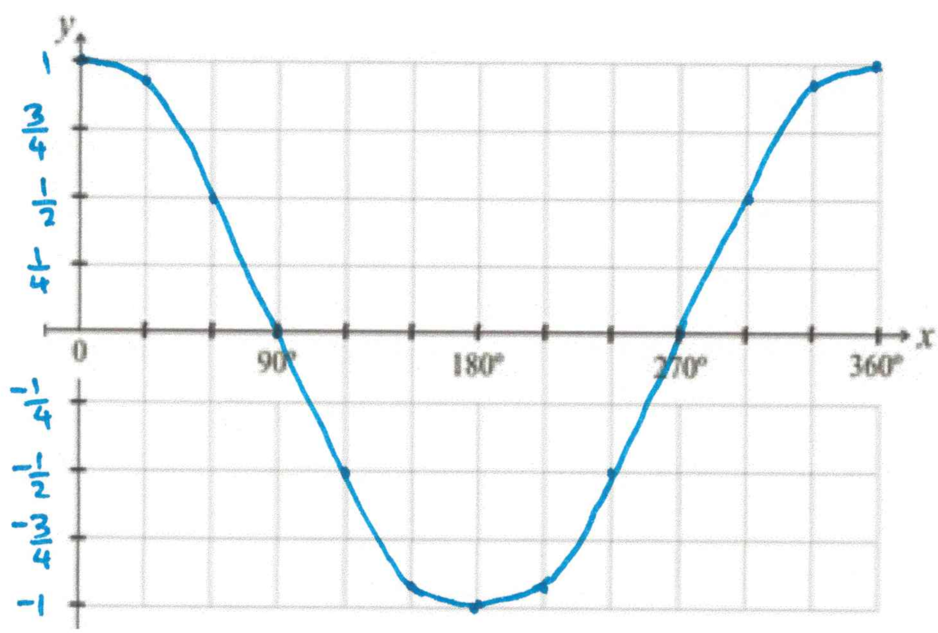
99. $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$ $-60°$ or $\frac{-\pi}{3}$

100. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$ $150°$ or $\frac{5\pi}{6}$

101. Thinking in terms of degrees, graph one period of the sine function on the interval $0^\circ \le x \le 360^\circ$, i.e. graph $f(x) = \sin(x)$. What is the amplitude?



102. Thinking in terms of degrees, graph one period of the cosine function on the interval $0^\circ \le x \le 360^\circ$, i.e. graph $g(x) = \cos(x)$.



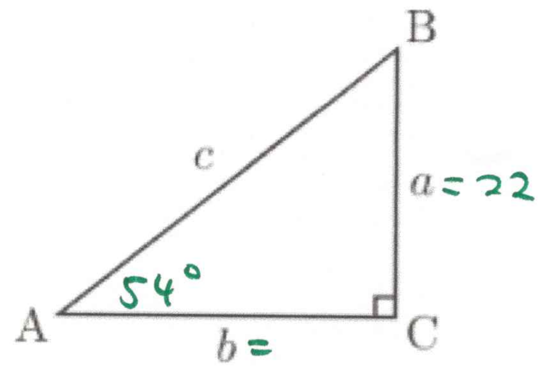
103. In right ΔABC, $a=22$ and $A=54^\circ$. Determine the length of side b. You may not use the Law of Sines or the Law of Cosines.

$$\tan 54^\circ = \frac{22}{b}$$

$$b \tan 54^\circ = 22$$

$$b = \frac{22}{\tan 54^\circ}$$

$$b \approx 15.984$$

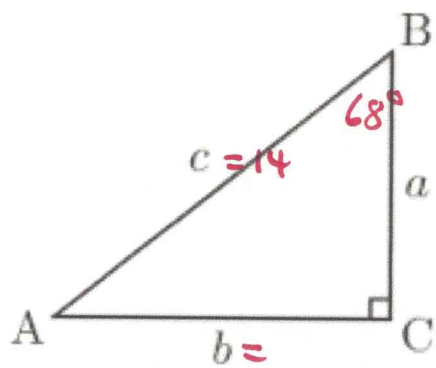


104. In right ΔABC, $c=14$ and $B=68^\circ$. Determine the length of side b. You may not use the Law of Sines or the Law of Cosines.

$$\sin 68^\circ = \frac{b}{14}$$

$$14 \sin 68 = b$$

$$12.981 \approx b$$



105. In right ΔABC, $c=40$ and $b=35$. Solve the triangle. You may not use the Law of Sines or the Law of Cosines.

$$\cos A = \frac{35}{40}$$

$$A = \cos^{-1}\left(\frac{35}{40}\right)$$

$$A \approx 28.955$$

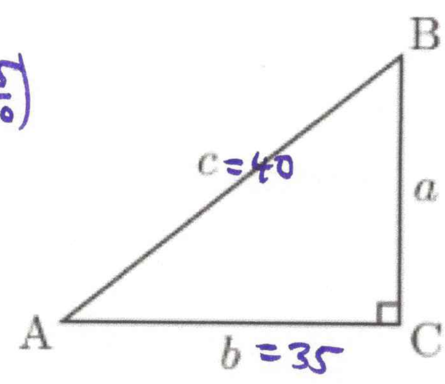
$$B \approx 90^\circ - 28.955 \approx 61.045$$

$$a^2 + 35^2 = 40^2$$

$$a^2 = 40^2 - 35^2$$

$$a = \sqrt{40^2 - 35^2}$$

$$a \approx 19.365$$



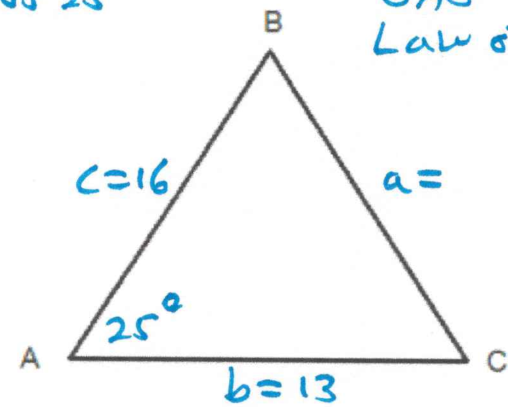
106. In ΔABC, $b=13$, $c=16$, and $A=25^\circ$. Determine the length of side a.

$a^2 = 13^2 + 16^2 - 2(13)(16)\cos 25^\circ$

$a^2 \approx 47.976$

$a \approx 6.926$

SAS
Law of Cosines



107. In ΔABC, $a=23$, $b=27$, and $c=31$. Determine the measure of angle B.

$b^2 = a^2 + c^2 - 2ac\cos B$

$27^2 = 23^2 + 31^2 - 2(23)(31)\cos B$

$27^2 - 23^2 - 31^2 = -2(23)(31)\cos B$

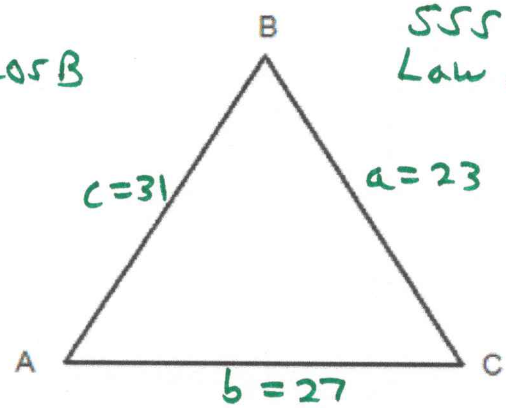
$\frac{27^2 - 23^2 - 31^2}{-2(23)(31)} = \cos B$

$0.5337 \approx \cos B$

$B \approx \cos^{-1}(0.5337)$

$B \approx 57.747^\circ$

SSS
Law of Cosines



108. In ΔABC, $a=15$, $c=13$, and $A=62^\circ$. Determine the measure of angle C.

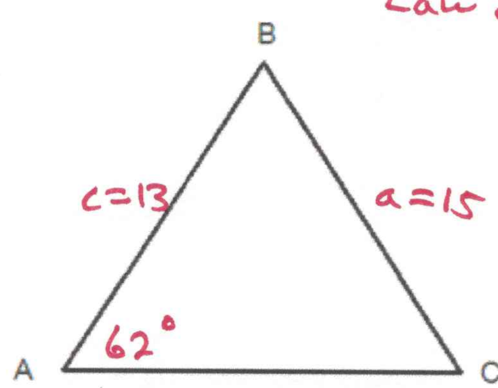
$\frac{\sin 62^\circ}{15} = \frac{\sin C}{13}$

$\frac{13\sin 62^\circ}{15} = \sin C$

$0.7652 \approx \sin C$

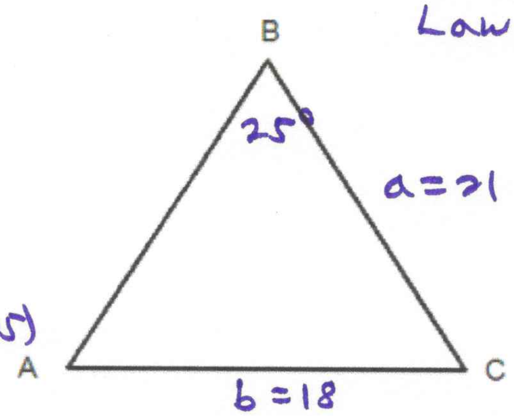
$C \approx 49.927^\circ$

SSA
Law of Sines



109. In ΔABC, $b = 18$, $a = 21$, and $B = 25^\circ$. Determine the two possible measures of angle A.

SSA
Law of Sines



$$\frac{\sin 25^\circ}{18} = \frac{\sin A}{21}$$

$$\frac{21 \sin 25^\circ}{18} = \sin A$$

$$0.49305 \approx \sin A$$

$$A \approx \sin^{-1}(0.49305)$$

$$A \approx 29.542^\circ$$

or

$A \approx 150.458$ supplementary angle to 29.542°